Australian Journal of Basic and Applied Sciences, 7(5): 266-277, 2013
ISSN 1991-8178

Modelling the Vertical Loads Applied by Pedestrians at a Range of Walking Velocities

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Abstract: Over the past half century numerous high profile bridges such as the Brooklyn Bridge, the London Millennium Bridge, and the Auckland Harbour Bridge have been subject to pedestrian induced vibrations due to walking. In this time various deferent guides have attempted to solve such problems, and in doing so produced models which conservatively estimate the loading of pedestrians. They were in large part left with little or no option as there is a scarcity of reliable information on the magnitude and nature of this type of loading. The authors have attempted to shed light on this by carrying out over 300 walking trials on 50 healthy adult participants walking at slow, normal, and fast velocities along a rigid walkway mounted with a force plate. Subject data, pertinent temporal-spatial parameters of gait, walking velocity and pacing frequency are presented. Additionally, the vertical forces recorded during these tests are presented and analysed. A single harmonic force function is therefore developed. This force function which incorporates a velocity dependent dynamic load factor offers an improvement over the guides, which present their dynamic load factors independent of gait parameters including pacing velocity.

Key words: pedestrians, vertical loading, walking, dynamic load factor, footbridges

INTRODUCTION

New slim-line and elegant footbridges are typically quite flexible and lightweight, rendering them sensitive to pedestrian induced vibrations vertically and/or laterally (perpendicular to the direction of bridge span). The lateral vibrations caused by crossing pedestrians that occurred on the high profile London Millennium Bridge in 2000 resulted in large research into vibrations in this specific direction (Wan, K., 2009; Zivanovic, S., A. Pavic, 2011). However, the vertical vibration of footbridges are a more common occurrence (Bocian, M., 2011). Even so, current guides on the issue of vertical vibrations induced by pedestrian loading have remained, largely, stagnant since BS 5400 was first introduced over 30 years ago (Ingolfsson, E.T., 2007). One of the main areas the guides fall short-in is that they conservatively overestimate the loading produced by pedestrians. This is principally to compensate for the fact that there is a current dearth of understanding of the complex dynamic loading generated by people walking on flexible surfaces. This study attempts to provide a better understanding of this complex loading regime through analysing the effects of various gait parameters and anthropometric data on vertical pedestrian loading.

Vertical Pedestrian Loading:

Walking imparts forces, GRFs, in three orthogonal planes; namely, a vertical in the coronal, a medial-lateral in the transverse, and a longitudinal in the mid-sagittal plane (Fig. 1). Of focus in this paper is the vertical force, the GRF of greatest magnitude and the one which will be discussed here on in. When a pedestrian walks he/she will produce two support phases during this gait/walking cycle (Fig. 2). These phases include the single support phase (when only one foot is on the ground) and the double support phase (when both feet are on the ground simultaneously, which is unique to walking). The two support phases cause two vertical GRF types; namely, a single foot force that acts as a component of the double support phase force (Kerr, S., 1998). The majority of guides published in the last decade present load models in the form of a single harmonic sine function (SHSF), hence considering human walking as periodic in nature:

$$F_v(t) = W + \alpha_v . W . \sin(2.\pi . f_s . t) k . z \text{ [N]} \quad (1)$$

The reference force, $\alpha_v . W$, although highly variable, is generally presented as a fixed value in the varies international guides. An improvement of this would, therefore, be to present such a reference force based on the characteristics of the population set that will use the bridge, such as their gait parameter relationship with vertical loading.

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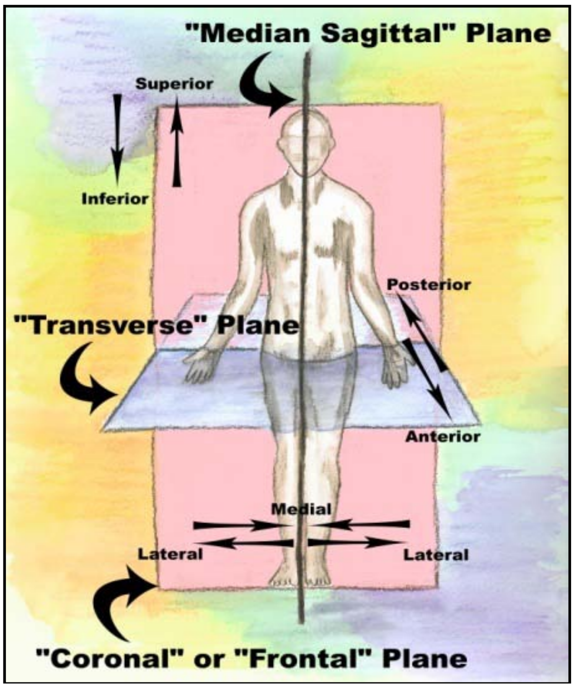

Fig. 1: The orthogonal planes of the human body (Neuro Surgical, 2011).

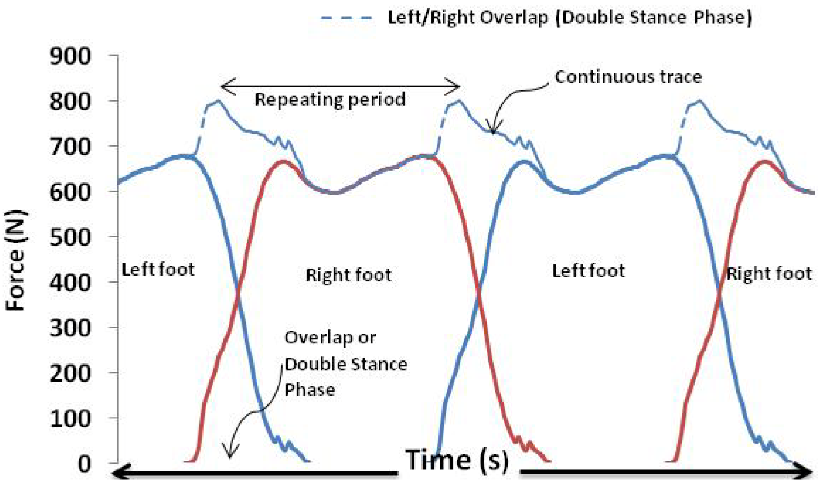

Fig. 2: Continuous walking vertical force.

Gait Parameters Pertinent to Vertical Pedestrian Loading:

Gait Parameters:

Although human locomotion is a very complex process, Davis *et al*. (2000) describe walking or 'gait' as a cyclic activity for which certain discrete events have been defined as significant. In biomechanical terms, these parameters are classed as either spatial or temporal and a list of these parameters is contained in Table 1. In terms of pedestrian loading on structures, the gait parameters of relevance depend on whether vertical or lateral excitation is being considered. Step length, pacing frequency, and the resultant vertical ground reaction force are the parameters which influence vertical loading.

Table 1: Spatial and temporal gait parameters.

Spatial Gait Parameters	Temporal Gait Parameters
Step Length	Pacing Frequency/Cadence
Step Width	Swing Phase
Foot Landing Position (Toe-in/Toe-out Angle)	Stance Phase/Support Phase
	Pacing Velocity

Step Length:

Step length, l_s, refers to the distance from the heel strike of one foot to the heel strike of the succeeding foot during forward gait. There is, however, some confusion in the literature between step length and stride length; e.g., Racic *et al*. (2009), Kirtley *et al*. (1985), and Henriksen *et al*. (2004) all imply the latter as the distance between the heel strike of one foot and the next heel strike of the same foot. Meanwhile others; including, Riley *et al*. (2001), Kerr (1998), and Pedersen and Frier (2010) refer to stride length as step length. Hence, to avoid confusion it may be best to avoid using the term stride length altogether and instead refer to right step length plus left step length as 'gait cycle length'. Some investigators claim that step length increases with age up to a point and then begins to diminish, as was evident in a review by Archbold and Mullarney (2010). That review [14] reports the mean step length to be 0.59m for children and young adults, 0.67m for mature adults, and 0.61m for persons over 60 years; with standard deviations of 8%, 11%, and 9%; respectively. Interestingly, Kirtley (2006) suggests the reason for this is that the taller you are the longer your step length will naturally be, hence females will have a reduced length in comparison to the taller on average male. As regards slow and fast paced walking step length values, there is a noticeable dearth of information in the literature. One of the relatively few reports is by Riley *et al*. (2001) where they list step length as 0.57 m for slow walking and 0.76 m for fast, with standard deviations of 7% and 11%, respectively.

Pacing Frequency:

Pacing frequency (foot-fall rate per second), f_s, is regarded as the most relevant of the gait parameters in terms of pedestrian loading on footbridges. Indeed, during a parametric study carried out by Archbold *et al*. (2011) it was found that pacing frequency has by far the most influence over vibration response in comparison to step length, mass, and leg stiffness. Moreover, low-frequency structures (such as footbridges) are primarily excited by repetitive footfalls when the pacing frequency is close to a structural resonance frequency (2011). Pacing frequency, itself, is defined as the inverse of the time taken from the initial contact of the left foot with the ground to the initial contact of the right foot immediately thereafter (or vice-versa) and corresponds to the rate of application of vertical forces. In biomechanical terms, this parameter is often measured as cadence (an old military term), which is the number of steps per minute rather than the number per second; possibly due to the fact it was traditionally easier to count the number of steps a pedestrian took in one minute rather than one second. The premise that steps per second equates to cycles per second (Hz) is based on the assumption that the load from each footstep is approximately the same and that the time taken for the feet to overlap is kept constant for a given pacing frequency, i.e., the load has a periodic nature (Pimentel, 2001; Fanning, 2005).

Keogh *et al*. (2010) reports an overall mean value of 1.96 Hz after reviewing seven papers. Notably, the majority of the results recorded in the literature are determined from trials conducted on rigid surfaces or gait machines. An exception to this is Zivanovic *et al*. (2005) who determined their mean of 1.87 Hz by analysing pedestrians crossing a bridge. Meanwhile, according to Kirtley's (2006) hypothesis (citing data from Sutherland (Rose, 1994) the taller you are the slower your pacing frequency is more likely to be for normal walking; he suggests that a person's leg will essentially act similar to the pendulum of a clock during walking. Pacing frequencies for slow and fast paced walking meanwhile, which are less well documented than normal paced walking, have been reported by Bilney *et al*. (2003) as 1.67 Hz for slow and 2.2 Hz for fast; with SDs of 8% and 4%; respectively

Pacing Velocity:

Pacing velocity, v_s, is defined as the distance travelled by a pedestrian in any direction over time; many reports unsurprisingly refer to it as walking speed. Intuitively, pacing velocity has a direct relationship with step length and pacing frequency. Kirtley (2006) proposes that pacing velocity increases linearly with pacing frequency, and logarithmically with step length. This implies that pacing velocity can be increased at lower velocities by quickening the pacing frequency and/or increasing the step length; however, at higher velocities it can only be increased by quickening the pacing frequency as maximum step length is reached well before maximum velocity.

A substantial review of 41 different papers was carried out by Bohannon and Williams Andrews (2011), which focused on the matter of normal walking for both males and females. In that review the mean values reported for males aged between 20 and 69 years ranged between 1.27 m/s and 1.55 m/s, and between 1.18 m/s and 1.48 m/s for females.

Bohannon and Williams Andrew's (2011) review highlights that walking velocity for both males and females is seemingly age related, i.e., from age 20 to 60 years walking velocity is roughly constant; but at about 60 years it begins to decline almost linearly with age. A second aspect of Bohannon and Williams Andrews (20111) review is that males tend to walk on average somewhat faster than their female counterparts, as their overall mean is 1.318 m/s (SD: 0.167 m/s) versus 1.262 m/s (SD: 0.157 m/s); respectively. However; Kasperski and Sahnaci (2007) report that males and females will tend to have the same pacing velocity where 'pace of life' is the dominant factor. This relates back to Kirtley's (2006) hypothesis (citing data from Sutherland *et al*.

(1994)) that shorter on average people; e.g., females in comparison to males, will increase their pacing frequency in order to compensate for their shorter on average step length. In contrast to normal paced walking, slow and fast has been reported as 1.11 m/s (SD: 16%) and 1.91 m/s (SD: 14%), respectively. However, these velocities are somewhat higher than the ones reported by Riley *et al*. (2001) who present values of 0.87 m/s (SD: 14%) for slow walking and 1.74 m/s (SD: 12%) for fast walking.

Experimental Programme:

The experimental programme reported herein consists of walking trials involving 15 female and 25 male healthy adult participants. The participants conducted the walking trials in the laboratory on a specially constructed rigid walkway as described in the following section.

Participants:

Participants were recruited from staff and students at AIT, Ireland. All were aged between 20 and 55 years. The ethnical composition of the participant sample was predominantly Caucasian with a small proportion being of African and Chinese background. Persons were excluded from participation if they had a history of previous injury with ongoing symptoms, or significant previous injury that would hamper their gait. All participants gave written consent according to the ethical procedures approved by AIT and its Research Ethics Committee.

Anthropometric Data:

The following parameters were recorded for each test participant prior to the walking trials being carried out: age; height (with and without footwear); weight. A summary of the recorded values is presented in Table 2.

Table 2: Age and anthropometric data for each gender group.

	Male		Female	
Parameter	Mean	SD	Mean	SD
Age (Year)	32.1	18.2	27.0	3.5
Height (m)(with footwear)	1.81	0.06	1.65	0.06
Weight (kg)	81.31	11.68	62.25	8.61

Equipment:

A rigid walkway was specially constructed to carry out the walking trials. The walkway is 0.9m wide x 11.0m long and is constructed from three 50mm thick laminated fibreboard panels framed with timber battens and cross members at 600mm centres, which were bolted together longitudinally and placed directly on the laboratory floor. A 500mm x 500mm AMTI AccuGait balance platform (force plate) was mounted at the mid-point of the walkway to record the ground reaction forces: the top surface of the force plate was made level with the top surface of the walkway. In the vertical direction, Fz, the force plate has a natural frequency of 150Hz and a loading capacity of 1334N and the force plate was calibrated prior to the walking trials through measurement of static forces. Three Monitran MTN1800 accelerometers, with a sensitivity of 1.020 V/g@80Hz, were mounted to the underside of the walkway at approximately one-third span, mid-span, and two-third span respectively.

Data were recorded from the accelerometers through a virtual instrument (VI) developed in National Instruments (NI) LabView 8.5. These data were used to determine the time interval between consecutive footsteps. Grid paper measuring 3.5m x 0.6m and containing a 20mm x 20mm grid size was placed over the middle section of the walkway to assist in recording the spatial parameters such as step length, step width and foot landing position from the trials. A schematic layout of the test set-up is shown in Fig. 3.

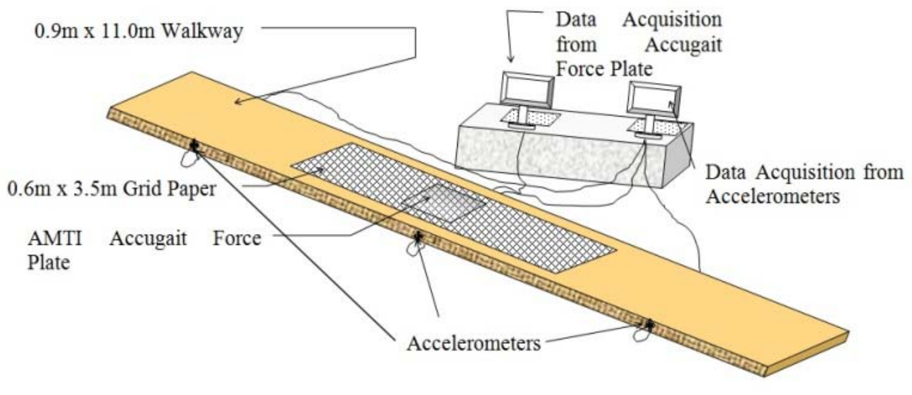

Fig. 3: Schematic representation of walkway and set up.

Experimental Procedure:

The participants were asked to wear their regular clothing and comfortable, flat-soled shoes for the walking trials. Prior to the recorded traversing of the walkway, each participant completed a number of ‘dummy’ runs to ensure they felt comfortable with the process. For these dummy trials and the actual walking trials, the test subjects were requested to walk in a straight line along the length of the walkway at their normal speed and gait, while looking straight ahead – this was aided through using visual targets on the facing walls (Fig. 4). Immediately prior to each trial the participant coated the soles of their shoes with blue chalk dust, which aided the recording of the footfall positions and thus measurement of the spatial gait parameters. This procedure has been successfully used by other authors (Taranto, 2005; McDonough, 2001; Wilkinson, 1997). In addition (Taranto, 2005) citing (Clarkson, 1983; Burnett, 1971; Levangie, 1990; Freychat, 1996; Shores, 1980) and by conclusion of their own experimental work suggests the footprint method of assessing gait parameters easy, reliable, valid, inexpensive and clinically feasible. Participants carried out an average of two slow velocity walks, three normal velocity walks, and then two fast velocity walks. Each velocity was achieved by requesting the participant to walk slower than they would normally do for slow walking, as if they were in a funeral procession or similar; faster than normal (without running) for fast walking, as if they were rushing to get somewhere; and the velocity they felt most comfortable at for normal walking.

Fig. 4: Walking trial in progress.

RESULTS AND DISCUSSION

Gait Parameters:

Step Length:

Presented in Fig. 5 and Fig. 6 and are the recorded step lengths for slow, normal, and fast walking. In Fig. 6 the histogram bin ranges from the shortest step value, which occurred during slow walking (0.28 m), to the longest step value, which occurred during fast walking (1.06 m), while a bin width of 0.02 m was chosen by inspection.

Perhaps the most interesting aspect of Fig. 6 is that step lengths for normal and fast paced walking are quite close to one another. In demonstration, the increase in mean step length from slow to normal paced walking for the entire group was 32%, in contrast to the 12% increase from normal to fast walking. This latter increase was found to be 9% for females and 17% for males, while the slow to normal increase was 27% and 34% for these two respective genders. This apparent gender difference may be related to the logarithmic relationship, noted by Kirtley (20096), between step length and pacing velocity; which implies that the shorter on average female will find it more difficult to increase their step length when increasing pacing velocity. Another interesting aspect of Fig. 6 is that the fast walking distribution is bi-model in profile. A possible reason for this may be due to differences in the shorter on average female being unable to increase her step length to the same extent as the taller on average male at fast velocities.

Pacing Frequency:

Represented in Fig. 7 and Fig.8 are the distributions and recorded pacing frequencies from the slow, normal, and fast walking trials. The respective means for each of the pacing frequencies are 1.43 Hz (SD: 14%), 1.89 Hz (SD: 10%), and 2.21 Hz (SD: 10%), which represents a 32% increase from slow to normal, and a 17% increase from normal to fast. In Fig.8 the histogram bin ranges from the minimum value for slow walking (0.66 Hz) to the quickest frequency for fast walking (3.04 Hz), while a bin width of 0.05 Hz was chosen by inspection.

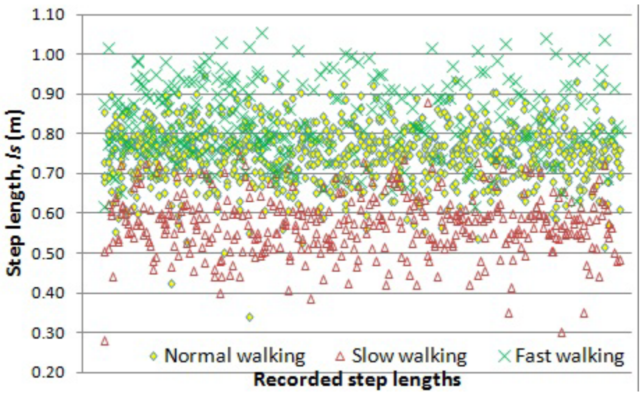

Fig. 5: Range of mean step lengths for slow, normal and fast walking.

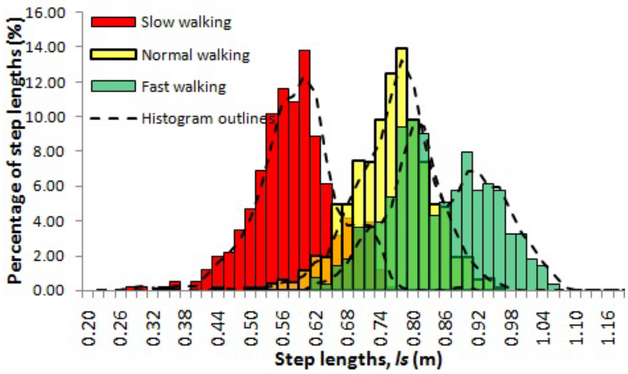

Fig. 6: Distribution of mean step lengths for slow, normal and fast walking.

The slow and fast paced frequencies compared with Bilney *et al*. (2003) results shows a 17% difference in terms of slow walking and only a 0.5% difference in terms of fast walking. Bilney *et al*. (2003) reports a value of 2.22 Hz for fast walking and a value of 1.67 Hz for slow. Unfortunately, other reports in the literature in terms of slow and fast paced walking are somewhat limited. Interestingly, during the trials the participants found walking slow to be the most difficult gait to carry out: they noted that walking slow is a gait that they generally refrain from. Furthermore, when asked to name any situations they might actually walk slowly they noted funeral processions and crowd walks as two of the more common answers.

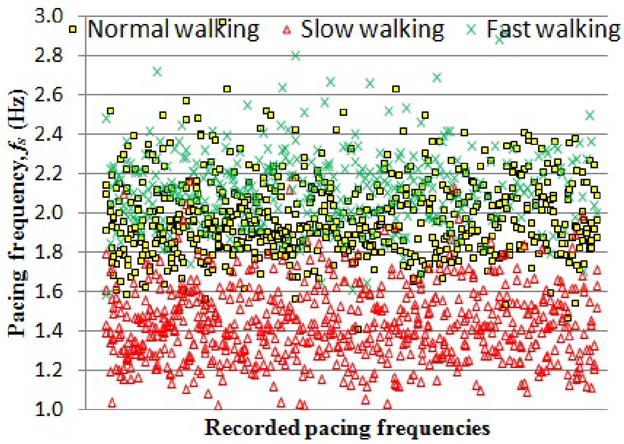

Fig. 7: Range of mean pacing frequencies for slow, normal, and fast walking.

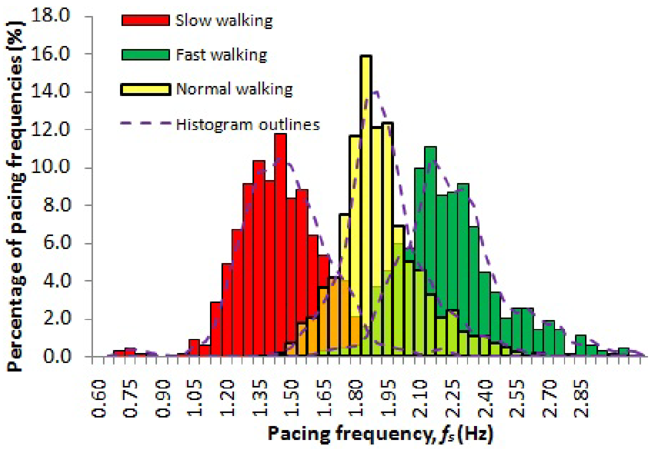

Fig. 8: Distributions of mean pacing frequencies for slow, normal, and fast walking.

Pacing Velocity:

Presented in Fig. 9 and Fig. 10 are the pacing velocities for the three walking trials; namely, slow, normal, and fast walking – each pacing velocity was determined via the product of mean pacing frequency and step length for each trial walk. An interesting aspect from Fig. 9 is that each pacing velocity category has its own unique velocity range. To add further, the majority of the slow walking velocities fall between 0.60 m/s and 1.25 m/s, normal walking 1.25 m/s and 1.65 m/s, and fast walking 1.6 m/s and 2.25 m/s. The three respective mean values for these velocities were 0.83 m/s, 1.43 m/s, and 1.90 m/s with accompanying SDs of 18%, 11%, and 10%; respectively. The former of these velocity values, slow walking, is only slightly less than Riley *et al*. (2001) reported value of 0.87 m/s (SD: 14%), while the latter is somewhat higher than Riley *et al*. (2001) fast walking reported mean of 1.74 m/s (SD: 12%) – unfortunately, further reports on slow and fast walking are somewhat lacking in the literature. In terms of pacing velocity gender differences Fig. 10 highlights that fast walking is the only real instance where this is present, as there is only a 2.4% difference in the overall gender mean for slow walking, a 1.4% difference for normal walking; but a 7.7% for fast walking. The reason for the fast walking difference may be due to the taller on average male having the ability to lengthen (or maximise) their step length greater than the shorter on average female when trying to increase their pacing velocity.

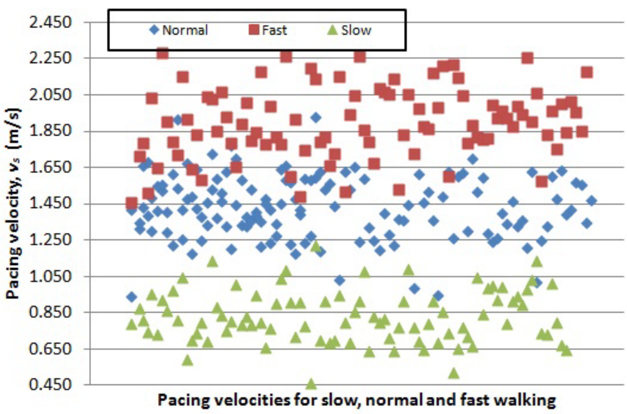

Fig. 9: Range of mean pacing velocities for slow, normal, and fast walking.

Force Modelling:

Vertical Dynamic Load Factor (DLF):

The vertical DLF is defined as the ratio of maximum increase in GRF from the static weight divided by the static weight of the pedestrian. The DLFs from the normal walking trials, therefore, are presented in Fig. 11 and Fig. 12. The histogram bin ranges from 0.05 to 1.35, as these two values are the minimum and maximum values, respectively; while the bin width is 0.05, which was chosen by inspection. The distribution is slightly skewed to the right, but the reason for this is unclear. For instance it is not clear if it is a gender issue, as Fig. 12 demonstrates that there is no real difference in gender mean values and ranges: as there is only a 2.0% difference between both gender groups with an overall mean 0.569 (SD: 0.239).

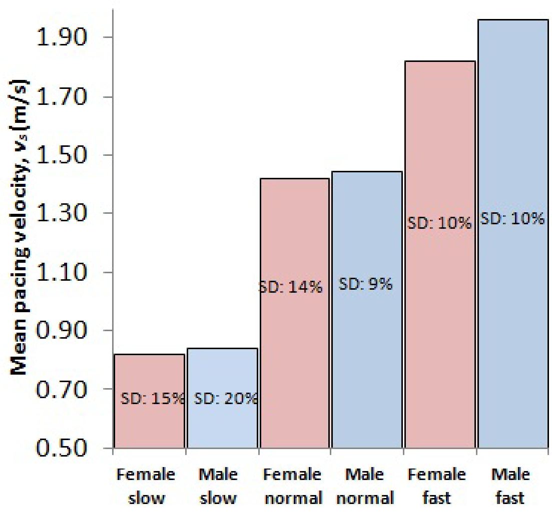

Fig. 10: Mean pacing velocities for slow, normal, and fast walking.

An interesting aspect of both Fig. 11 and Fig. 12 is that approximately 80% of the DLFs range between 0.25 and 1.10. This suggests the majority of pedestrians walking at normal velocities will have a DLF in the latter range. Each individual DLF was determined by assuming each footstep to be identical to the next, and was the maximum force once left and right foot step forces were summated; the time interval between each heel strike was determined by inverting the pacing frequency for each trial walk.

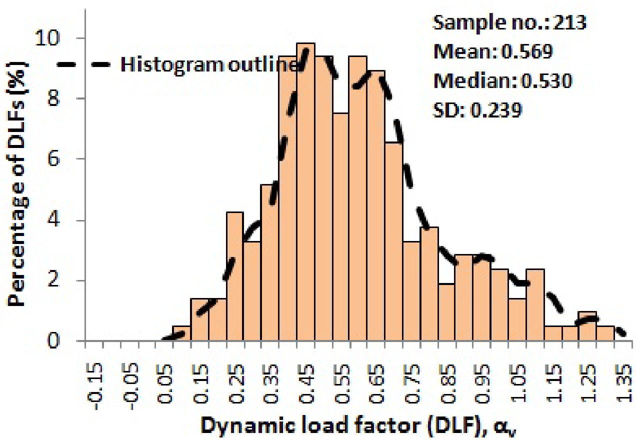

Fig. 11: Distribution graph of DLFs for normal walking.

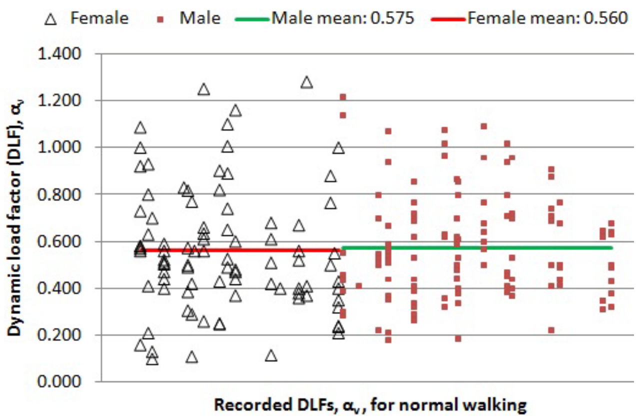

Fig. 12: Range of recorded DLFs for normal walking.

Relationship Between Single Footfall DLF and Pacing Frequency:

Comparing the overall mean DLF for the single footfall (SF) force with that of the overall force suggests the latter is roughly equal to 2.5 times the former. In view of this, if the SF force could be predicted/modelled accurately the maximum force could then be determined. Presented in Fig. 13 and Fig. 14, therefore, are the relationships between the pacing frequencies and the vertical SF DLFs determined during the three walking trial sets; namely, slow, normal, and fast walking. The Fig. 14 illustrates quite a poor relationship between the vertical DLFs and the walking frequencies when plotted against each individual walking category. This is even more apparent once the low R^2 values are realised with 0.16 resulting for slow, 0.11 for normal, and 0.04 for fast walking. Interestingly, however, the relationship is vastly improved once the three walking categories are combined and plotted against the vertical SF DLF values, as the R^2 value of 0.57 testifies. This is similar in form to that previously proposed in the literature; however, the magnitude is considerably lower. For example, at a pacing frequency of 1.9 Hz, this relationship estimates a dynamic load factor of 0.227, as opposed to 0.375 and 0.399 which would have been obtained previously using Archbold (2004) and Ells' (2000) models; respectively.

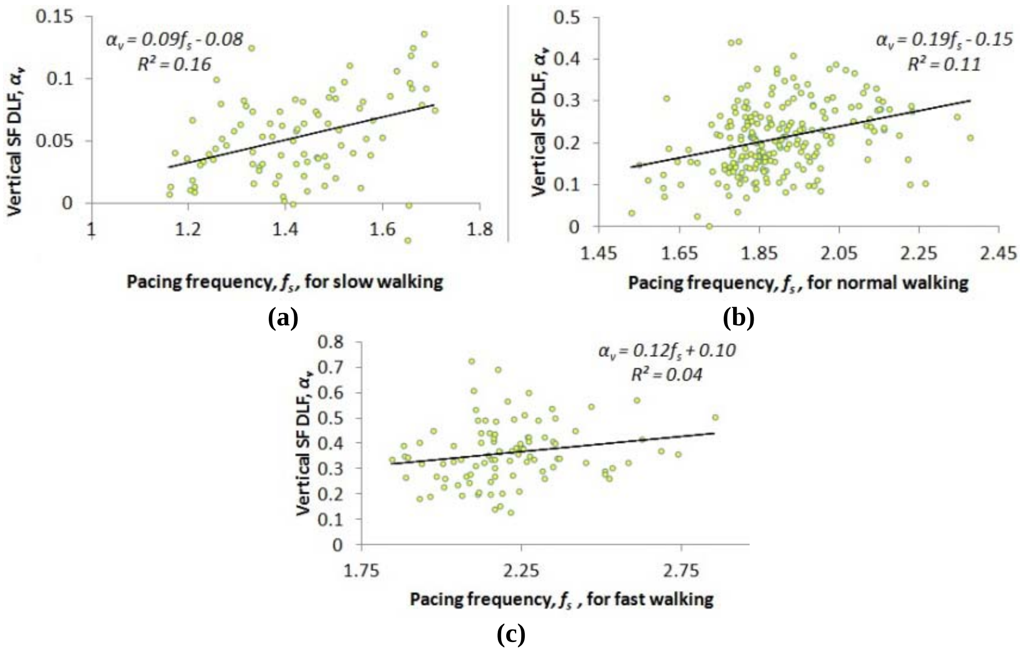

Fig. 13: SF Vertical DLFs plotted against (a) slow, (b) normal, and (c) fast walking pacing frequencies.

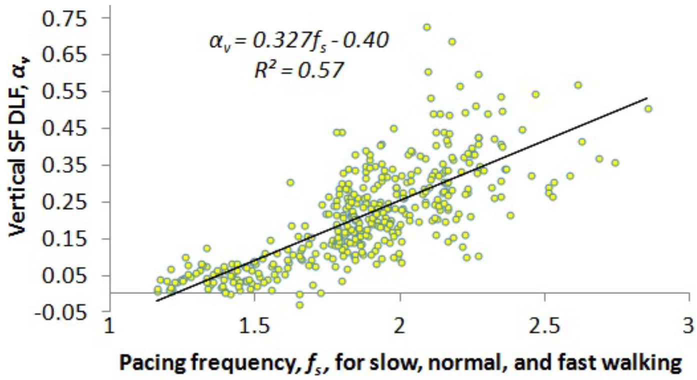

Fig. 14: SF Vertical DLFs plotted against slow, normal, and fast walking pacing frequencies combined.

Relationship Between Single Footfall DLF and Pacing Velocity:

Previously (Archbold, 2011) the authors found that pacing velocity was more closely linked to vertical loading than pacing frequency was for normal walking (Fig. 15). This claim was also supported by Kirtley (2006) who reported a relationship between ranges 1.0 m/s and 1.65m/s.

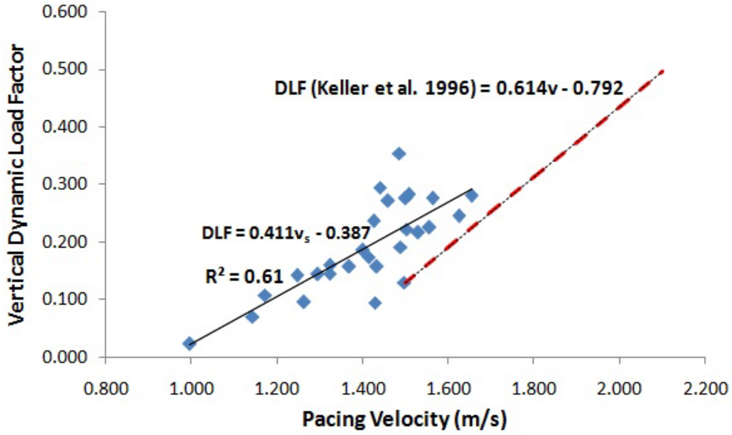

Fig. 15: Relationship between SF DLF and pacing velocity for normal walking (37).

In view of the relationship in Fig. 15, the vertical SF DLF versus all three velocities (slow, normal, and fast walking) is plotted; Fig. 16. The plot itself has a polynomial relationship. For instance, at velocities below approximately 0.5 m/s the vertical SF DLF is roughly equal to zero, this DLF then starts to gradually increase between velocities 0.6 m/s and 1.5 m/s, while between 1.5 m/s and above the gradient of the curve then becomes steeper while at the same time truly linear. The latter suggests that when a pedestrian starts to increase his/her velocity above 1.5 m/s the vertical DLF will be effected significantly; indeed, judging by the R^2 value of 0.75 pacing velocity has a direct influence on the single vertical SF DLF throughout. The only real section of the plot that seems to be skewed somewhat is where the plot moves above 2.00 m/s, as the data points become somewhat 'turbulent' in nature above this velocity. This perhaps relates to Taylor *et al*. (2004) hypothesises that walking fast is not a faster version of walking normal or slow, in that it is a different from of gait. This may imply, for some pedestrians; that above 2.00 m/s walking has reached the transition stage between walking and running, and for this reason the vertical SF DLF is influenced by more than just walking velocity.

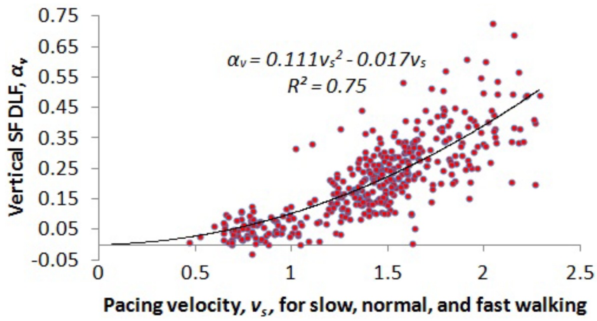

Fig. 16: SF Vertical DLFs plotted against slow, normal, and fast walking pacing velocities combined.

The improved relationship between velocity and the vertical DLF becomes logical once step length and the vertical DLF is investigated (Fig. 17). Although neither pacing frequency and step length individually offer a clearly defined relationship a combination of both (i.e. pacing velocity) do.

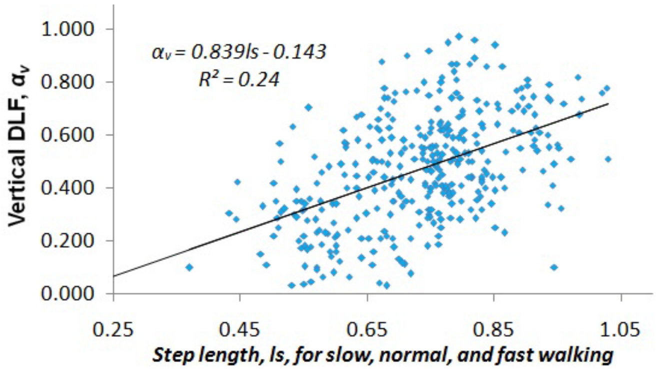

Fig. 17: SF Vertical DLFs plotted against slow, normal, and fast walking step lengths.

Therefore, from the relationship in Fig. 16 and by the virtue of fact the maximum dynamic force can be approximated to be 2.5 times the single stance phase force the DLF can be determine via Eq. 2. Hence, from Eq. 1 and Eq. 2, Eq. 3 is developed:

$$\alpha_v = (0.111 v_s{}^2 - 0.017 v_s) 2.5 \quad (2)$$

hence, $$F_v(t) = 2.5\,(0.111 v_s{}^2 - 0.017 v_s).W.\sin(2.\pi.f_s.t)\ [\mathrm{N}] \quad (3)$$

Conclusion:

A thorough investigation on vertical pedestrian loading was carried out in this paper. The results from the walking trials were analysed and presented; and secondly, from these results a force function was developed. This force function incorporates a vertical DLF which is dependent on pacing velocity, therefore offering a vast improvement on the current models provided which provide DLFs as fixed values; and therefore independent of pacing velocity and other gait parameters.

REFERENCES

Archbold, P., 2004. Novel Ineractive Load Models for Pedestrian Footbridges, PhD Thesis, in: School of Architecture, Landscape & Civil Engineering, University College Dublin, Dublin.

Archbold, P., B. Mullarney, 20011. The influence of selecteced gait parameters on vertical pedestrian loading, in: J. Biliszczuk, J. Bien, P. Hawryszkow, T. Kaminski (Eds.) Proceedings of the 4th International Footbridge Conference: Footbridge 2011-Attractive Structures at Reasonable Costs, 6-8 July, Dolnoslaskie Wydawnictwo Edukacyjne, Wroclaw, Poland, pp: 759-768.

Archbold, P., B. Mullarney, 2010. A review of gait parameters pertinent to pedestrian loading, in: N.A. Ni Nuallain, D. Walsh, R. West (Eds.) The 5th Joint Symposium Proceedings of Bridge & Infrastructure Research in Ireland, and Concrete Research in Ireland, 2-3 September, Cork, Ireland, pp: 25-34.

Archbold, P., J. Keogh, C.C. Caprani, P. Fanning, 2011. A parametric study of pedestrian vertical force models for dynamic analysis of footbridges, in: C. Gentile, F. Benedettini (Eds.) Proceedings of the 4th Internatioanl Conference on Experimental Vibration Analysis for Civil Engineering Structures: EVACES 2011, Verenna, Italy, pp: 339-346.

B.S.I., BS 5400:2; Steel, Concrete and Composite Bridges - Part 2: Specification for Loads, in: Section 7: Footway and cycle track live loads, Appendix C: vibration serviceability requirements for foot and cycle track bridges, British Standards Association, London, U.K., 1978.

Bilney, B., M. Morris, K. Webster, 2003. Concurrent related validity of the GAITRite® walkway system for quantification of the spatial and temporal parameters of gait, Gait & Posture, 17: 68-74.

Bocian, M., J. Macdonald, J. Burn, 2011. Modelling of self-excited vertical forces on structures due to walking pedestrians, in: G. De Roeck, G. Degrande, G. Lombaert, G. Muller (Eds.) Proceedings of the 8th International Conference on Structural Dynamics, Eurodyn 2011, 4-6 July, Leuven, Belgium, pp: 1110-1117.

Bohannon, R.W., A. Williams Andrews, 2011. Normal walking speed: a descriptive meta-analysis, Physiotherapy, 97: 182-189.

Burnett, C.N., E.W. Johnson, 1971. Development of gait in childhood: Part II, Developmental Medicine & Child Neurology, 13: 207-215.

Clarkson, B.H., 1983. Absorbent paper method for recording foot placement during gait: suggestion from the field, Physical Therapy: Journal of The American Physical Therapy Association, 63: 345-346.

Davis, R.B., P.A. DeLuca, S. Oumpuu, 2000. The Biomedical Engineering Handbook, 2nd ed., CRC Press LLC.

Ellis, B.R., 2000. On the response of long-span floors to walking loads generated by individuals and crowds, The Structural Engineer, 78: 17-25.

Fanning, P., P. Archbold, A. Pavic, 2005. A novel interactive pedestrian load model for flexible footbridges, in: Proceedings of the Conference & Exhibition on Experimental & Applied Mechanics, Society for Experimental Mechanics; SEM 2005, Portland, Oregon, USA.

Freychat, P., A. Belli, J.P. Carret, J.R. Lacour, 1996. Relationship between rearfoot and forefoot orientation and ground reaction forces during running, Medicine & Science in Sports & Exercise, 28: 225.

Henriksen, M., H. Lund, R. Moe-Nilssen, H. Bliddal, B. Danneskiod-Samsøe, 2004. Test-retest reliability of trunk accelerometric gait analysis, Gait & Posture, 19: 288-297.

Ingolfsson, E.T., C.T. Georgakis, J. Jonsson, F. Ricciardelli, 2007. Vertical footbridge vibrations: towards an improved and codifiable response evaluation, in: A. Zingoni (Ed.) Proceedings of The 3rd International Conference on Structural Engineering, Mechanics, and Design: Recent Developments in Structural Engineering, Mechanics, and Design, Cape Town, South Africa, pp: 65-67.

Ingolfsson, T.E., 2011. Pedestrian-induced Lateral Vibrations of Footbridges: Experimental Studies & Probablistic Modellings, PhD Thesis. Published as DTU Civil Engineering Report R-231, January 2011, in: Department of Civil Engineering, Technical university of Denmark, UK, 2011.

Kasperski, M., C. Sahnaci, 2007. Serviceability of pedestrian structures, in: Proceedings of the International Modal Analysis Conference (IMAC XXC), Orlando, USA.

Keogh, J., P. Archbold, C. Caprani, P. Fanning, 2010. Estimating the effect of pacing frequency on the vertical response of a flexible footbridge, in: N.A. Ni Nuallain, D. Walsh, R. West (Eds.) The 5th Joint Symposium Proceedings of Bridge & Infrastructure Research in Ireland, and Concrete Research in Ireland, Cork, Ireland, pp: 35-42.

Kerr, S., 1998. Human Induced Loading on Staircases, PhD Thesis, in: Department of Mechanical Engineering, London University, England, UK.

Kirtley, C., 2006. Clinical gait analysis: theory and practice, Elsevier Churchill Livingstone, Edinburgh.

Kirtley, C., M.W. Whittle, R.J. Jefferson, 1985. Influence of walking speed on gait parameters, Journal of Biomedical Engineering, 7: 282-288.

Levangie, P.K., J. Brouwer, S.H. McKeen, K.L. Parker, K.A. Shelby, The effects of the standard rolling walker and two posterior rolling walkers on gait variables of normal children, Physical & Occupational Therapy in Pediatrics, 9: 19-31.

McDonough, A.L., M. Batovia, S. Kwon, J. Ziai, 2001. The validity and reliability of the GAITrite system's measurement: a prelimainary evaluation, Archives of Physical Medicine & Rehabilitation, 82: 419-425.

Neuro Surgical, Anatomic Terminology Image, in, 2011.

Pedersen, L., C. Frier, 2010. Sensitivity of footbridge vibrations to stochastic walking parameters, Journal of Sound and Vibration, 329: 2683-2701.

Pimentel, R.L., A. Pavic, P. Waldron, 2001. Evaluation of design requirements for footbridges excited by vertical forces from walking, Canadian Journal of Civil Engineering, 28: 769-777.

Racic, V., A. Pavic, J.M.W. Brownjohn, 2009. Experimental identification and analytical modelling of human walking forces: Literature review, Journal of Sound and Vibration, 326: 1-49.

Riley, P.O., U.D. Croce, D. Casey Kerrigan, 2001. Propulsive adaptation to changing gait speed, Journal of Biomechanics, 34: 197-202.

Rose, J., J.G. Gamble, V.T.H.W. Inman, 1994. Human walking, 2nd ed / edited by Jessica Rose, James G. Gamble. ed., Williams & Wilkins, Baltimore; London.

Shores, M., 1980. Footprint analysis in gait documentation: an instructional format, Physical Therapy: Journal of The American Physical Therapy Association, 60: 1163-1167.

Sutherland, D.H., K.R. Kaufman, M.P. Wyatt, S.J. Mubarak, 1994. Effects of botulinum toxin on gait of patients with cerebral palsy: Preliminary results, Gait & Posture, 2: 52-52.

Taranto, J., M.J. Taranto, A. Bryant, K.P. Singer, 2005. Angle of gait: a comparative reliability study using footprints and the EMED-SF®, The Foot, 15: 7-13.

Taylor, A.J., H.B. Menz, A.M. Keenan, 2004. The influence of walking speed on plantar pressure measurements using the two-step gait initiation protocol, The Foot, 14: 49-55.

Wan, K., S. Zivanovic, A. Pavic, 2009. Design spectra for single person loading scenario on footbridges, in: Proceedings of the IMAC-XXVII, Orlando, Florida, USA, 9-12 February.

Wilkinson, M.J., H.B. Menz, 1997. Measurement of gait parameters form footprints: a reliability study., The Foot, 7: 19-23.

Zivanovic, S., A. Pavic, 2011. Quantification of dynamic excitation potential of pedestrian population crossing footbridges, Shock and Vibration, 18: 563-577.

Zivanovic, S., A. Pavic, P. Reynolds, P. Vuiovic, 2005. Dynamic analysis of lively footbridges under everyday pedestrian traffic., in: Proceedings of the 6th International Conference on Structural Dynamics, EURODYN, Paris, France.