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The relationship between pedestrian loading and dynamic response of an FRP composite footbridge

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Abstract. Pedestrian loading on flexible structures such as footbridges, grandstands and lightweight floors is an area, which is receiving significant attention from the research community of late. One of the key parameters in determining the structural response is the frequency of the bridge.

The authors are currently researching pedestrian-induced loading on flexible structures and also the use of FRP materials in construction. This paper describes the amalgamation of these two discrete research interests by detailing the material testing, design and construction of a laboratory-scale FRP composite footbridge.

The bridge was constructed from glass fibre reinforced polymer (GFRP) composite beams, with GFRP lateral bracing. This structure supports a timber deck. The bridge is lightweight and the span can be altered from 6.5 m to 8.0 m clear span to adjust the structural response, by altering the natural frequency and magnitudes of displacements. The bridge can also be fixed in position through the use of removable intermediate supports. The bridge also has a force plate mounted at mid-span, facilitating direct measurement of the reaction force between the pedestrian and the structure.

This paper presents the results of preliminary walking trials on this bridge in both the fixed and free suspension states, and across a range of spans, allowing analysis of the interaction between pedestrian loading and the structural response of flexible structures.

Keywords: Human-structure interaction, FRP composite, vibration, pedestrian, footbridge

1. Introduction

Modern structural design tends towards shallower construction depths for horizontal members such as floor slabs, beams and bridges. This trend is driven by practical reasons – reduced floor depths giving increased ceiling heights or reduced overall building heights for high-rise structures – and for aesthetic reasons – one of the main drivers in relation to bridge design. Advances in construction technology, structural materials and ever-increasing structural analysis capability has facilitated this trend. Consequently, long-span structures such as footbridges and stadia grandstands, where the imposed static actions due to pedestrians are relatively small, might exhibit dynamic behaviour marked by closely-spaced natural frequencies and/or frequencies very close to the values perceived by human beings [1].

As a result of this trend, particularly for these types of structures, serviceability performance becomes increasingly more critical than in heavier construction. Due to their relative flexibility and associated low natural frequencies, the types of transient loading applied to these structures often elicits unexpected dynamic responses. Some high profile examples of structures exhibiting excessive dynamic behavior resulting from applied loads include the Millennium Bridge, London [2], the Singapore Airport's Changi

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Mezzanine Bridge [3], the Clifton Suspension Bridge in Bristol, UK [4], and the Pedro e Inês Footbridge in Portugal, [5]. Even more recently, in New York, the Squibb Park Bridge in Brooklyn closed in 2014 and remains closed pending remedial work to reduce the vibrations induced by pedestrian loading [6]. While the actual response of the aforementioned bridges varied, the common factor between them all was the vibration response caused by pedestrian loading. In relation to human-induced vibration, footbridges and long-span floors present significant challenges as the pursuit of lighter, more flexible structures continues. Numerous approaches have been developed in international design codes to address this problem.

1.1. Design approaches for structures subjected to human loading

In general there are two approaches within design guidelines towards dealing with footbridge vibration serviceability. The first approach requires the designer to avoid designing footbridges that have specific natural frequencies and is referred to as a '*Frequency Tuning Approach*'; i.e., the bridge is designed to avoid certain frequencies. While the second allows footbridges to be designed within specific frequency ranges as long as the dynamic response of the bridge is within acceptable limits for bridge users, and may be considered as a '*Vibration Limit Approach*'; i.e., the dynamic response of the bridge to a dynamic load model is analysed.

The Frequency Tuning Approach is obviously restrictive as structures with natural frequencies in the range of concern are prohibited, without further analysis of their actual dynamic performance. For this reason, many of the codes now allow for a Vibration Limit Approach to designing the structures with frequencies within this range.

1.2. Crowd loading enhancement factors

One of the major shortfalls, however, of the Vibration Limit Approach is that there is not presently a consensus on the optimum method for simulating pedestrian loading on flexible structures. Most methods specify a load model to represent a single pedestrian as either a moving or a stationary cyclical force. The overall crowd effect is thus obtained by multiplying the single-pedestrian response by a number to represent the amount of the crowd walking in phase. There is considerable disagreement on the value of this crowd multiplier number or enhancement factor. Figure 1 represents the discrepancies between four commonly adopted values [7, 8, 9, 10] for this enhancement factor (which aims to represent the actual number of pedestrians in a crowd who are walking in phase and at the same pacing frequency).

In addition to varying approaches to determining the crowd enhancement factors, there are numerous strategies for modelling the effect of the single pedestrians. The most common methods are summarized hereunder.

1.2.1. Moving point force model

This is the most common model used for simulating the vertical forces applied by a walking pedestrian. This model considers the pedestrian load as a moving, sinusoidal force, with a static component equal to the pedestrian's weight and an associated dynamic component, which is determined by the pacing frequency of the person. In general terms, the force can be represented as:

$$F(t) = G + r_1 G \sin(2\pi f_s t) \quad (1)$$

where:

F(t) is the forcing function; G is the weight of the person crossing; r_1 is the dynamic force component factor, which is dependent on the actual pacing frequency; f_s is the pacing frequency (Hz); t is the time of load application. This version only includes the pacing frequency of the pedestrian, which is chosen to match the first vertical natural frequency of the bridge to give a worst-case scenario. Several researchers argue that higher harmonics of this frequency should also be considered but the evidence for this is unclear.

This represents a cyclical sine wave load model, which is moved across the span of the bridge. However, this model does not take the mass of the person walking into account and tends to be conservative in its prediction of vertical accelerations.

1.2.2. Moving mass and dynamic component model

Fanning et al. [11] further refined this model to include the mass of the pedestrian and a dynamic component. However, the authors showed that, while this improved the accuracy of the predicted mid-span accelerations from a single person traversing a footbridge, it also tended to overestimate the actual measured vertical response.

Figueiredo [12] incorporated a factor into a similar model to account for the force exerted by the heel strike on the bridge as proposed by Varela [13],

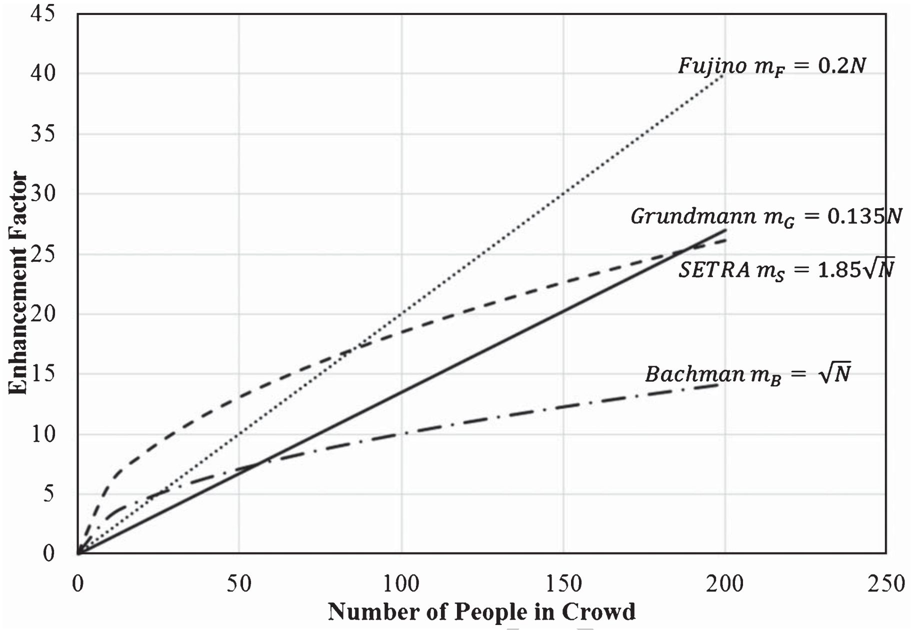


Fig. 1. Proposed Enhancement Factors for Vertical Loading Due to Crowd Synchronisation.

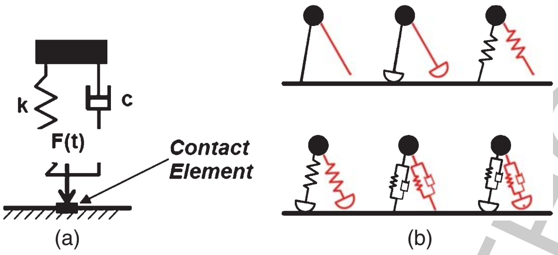


Fig. 2. SDOF Spring Mass Damper (a) and Inverted Pendulum Models (b) [15].

with no significant improvement on the predicted mid-span vibration levels.

1.2.3. Spring mass damper interactive model

Neither of the above load models accounted for any interaction between the crossing pedestrian and the vibrating structure. In an attempt to address this, Archbold proposed a model whereby the person was modelled as a vertical spring-mass-damper single degree of freedom system, as shown in Fig. 2(a). This model was given a stiffness which is similar to reported leg stiffness values and the mass of the pedestrian was applied and subjected to a dynamic force component as above. This model provided extremely accurate predictions of mid-span accelerations in the case of a footbridge which was subjected to 100 individual pedestrian crossings [14].

1.2.4. Inverted pendulum models

Stemming from the field of biomechanics and owing to the bipedal nature of human walking, considerable effort has been devoted to modelling human locomotion using an inverted pendulum model. These models allow for control mechanisms to be included which can take cognizance of physiological and psychological factors related to humans' balancing control. Dang summarized the various formats of these inverted pendulum models as shown in Fig. 2(b) [15].

It is evident from the approaches above that greater understanding of the interaction between vibrating structures and the traversing pedestrians is imperative to further the accuracy of simulation methods.

1.2.5. Stochastic crowd load models

More recently there has been a trend to move away from deterministic models such as those mentioned above and increased emphasis has been placed on trying to develop stochastic models of crowd loading, which will better represent both inter- and intra-pedestrian variability [16–18]. Many of these models employ stochastic distributions of gait parameters such as pacing frequency and step length, together with anthropometric properties such as the weight and height of the pedestrians. There is little focus on the magnitude of the forces applied to the vibrating surface – further research in this area is necessary.

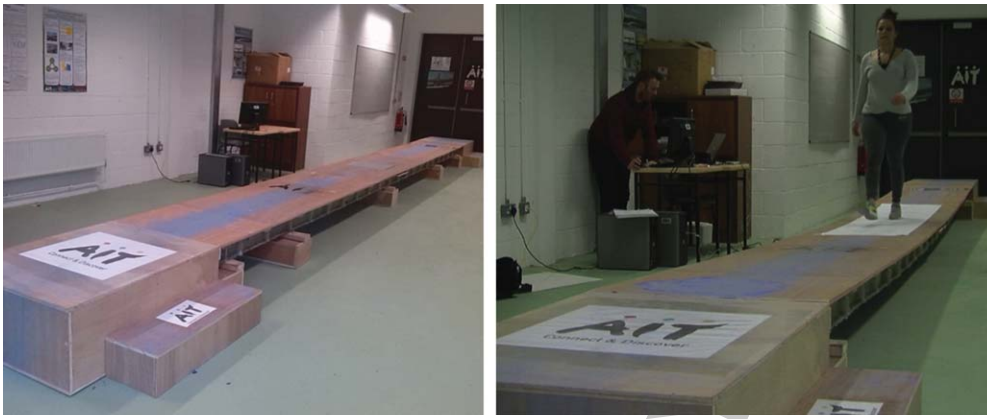

(a) (b)

Fig. 3. (a) Laboratory Scale Test Footbridge, (b) Walking trial in progress.

This paper reports on a study carried out by the authors to further understand the nature of vertical loading applied by pedestrians on a flexible structure, with a vertical natural frequency in the range susceptible to excitation by human walking. This work was carried out on a specially designed and constructed laboratory scale footbridge.

2. Laboratory-scale test bridge

In order to gain a better understanding of the loading applied by pedestrians to a flexible structure, the authors developed a unique laboratory-scale footbridge. The bridge was designed to have a natural frequency in the range which is excitable by human walking, to be lightweight and flexible enough to be excited by a single pedestrian, and to allow for various walking and load configurations.

The bridge (Fig. 3) is constructed from glass fibre reinforced polyester (GFRP) beams, which were fabricated by the authors. These beams support a plywood deck. The bridge span can be altered from 6.50 m to 8.0 m due to the flexible supports and there is a portable force plate built into the bridge deck. There are access and egress platforms, giving a total walking surface length of 11.0 m. The bridge can also be configured to act as a rigid platform by including intermediate supports. This allows comparison between the behaviours and associated walking forces on both rigid and flexible surfaces. The inclusion of the force plate in the bridge deck allows direct measurement of the forces imparted from the crossing pedestrian to the bridge surface. This presents a unique advance on other reported methods, whereby the forces are estimated through regression analysis from the bridge response.

The mass of the bridge was calculated to be 27.8 kg/m run, and the flexural modulus was determined to be 15.5 GPa. With an 80 kg mass (close to that of an average pedestrian) placed at the midspan; the bridge was found to have a natural frequency of 2.18 Hz, 2.34 Hz, 2.58 Hz, and 2.97 Hz for the flexible spans of 8.0 m, 7.5 m, 7.0 m and 6.5 m – respectively. Further details on the design and construction of this bridge are presented elsewhere [19].

3. Experimental programme

The aim of the work reported herein was to directly compare the spatio-temporal gait parameters and associated vertical ground reaction forces for individual pedestrians crossing the test structure under a number of configurations. Changing the bridge span and intermediate support conditions allows for study of the effect of bridge response on the loading applied.

The sample of test subjects (N = 26) is comprised of healthy adults, (age = 29.2 ± 5.6), with no history of leg or hip injury. The group represents both males (N = 18) and females (N = 8).

3.1. Participants

Participants were recruited from staff and students at Athlone Institute of Technology, Ireland. The ethnical composition of the participant sample was predominantly Caucasian with a small proportion being of Chinese background. Persons were excluded from participation if they had a history of previous injury with ongoing symptoms, or significant previous injury that would hamper their gait. In total there were 284 walks conducted: 112 on the rigid platform and 172 on the flexible platform. The participants were asked to walk at designated perceived walking speeds of slow, normal and fast. Trials were carried out on both the rigid configuration and the 4 available spans (6.5 m, 7.0 m, 7.5 m and 8.0 m). Participants took part in various combinations of these walking speeds and bridge configurations. Generally, each trial for each test subject was carried out 3 times.

3.2. *Anthropometric data*

The following parameters were recorded for each test participant prior to the walking trials being carried out: age; height (with footwear); weight. A summary of the recorded values is presented in Table 2.

3.3. *Data acquisition*

For both the rigid and flexible trials the same 500 mm × 500 mm AMTI AccuGait balance platform (force plate) was mounted at the mid-point of the walkway to record the ground reaction forces: the top surface of the force plate was made flush with the top surface of each walkway. The force plate is capable of measuring forces and moments in three orthogonal planes. Prior to dynamic testing, the force plate was calibrated through measurement of static forces. Monitran MTN1800 accelerometers, with a sensitivity of 1.020 V/g@80 Hz, were mounted to the side of the walkway at midspan and 1/3 spans (Fig. 4). The trial participants' pacing frequency during the flexible walking trials were determined via video analysis; this method was calibrated during the rigid trial walks against the accelerometer readings. Grid paper measuring 4.2 m × 0.6 m and containing a 20 mm × 20 mm grid size was placed over the middle section of the walkway to assist in recording the spatial parameters such as step length, step width, foot landing position.

3.4. *Experimental procedure*

The participants were asked to wear their regular clothing and comfortable, flat-soled shoes for both walking trial programmes. Prior to the recorded traversing of the walkways, each participant completed a number of "dummy" runs to ensure they felt comfortable with the process. For these dummy trials and the actual walking trials, the test subjects were requested to walk in a straight line along the length of the walkway at a normal speed, while looking straight ahead – this was aided through using visual targets on the facing walls. Immediately prior to each trial the participant coated the soles of their shoes with blue chalk dust, which aided the recording of the footfall positions and thus measurement of the spatial gait parameters. This procedure has been successfully used by other authors [20–22]. Each test subject completed a minimum of three recorded trials on each walkway. Figure 5 offers a diagrammatic representation of the test arrangement.

4. Results

The results from all of the walking trials have been processed to determine the salient information in relation to the influence of the vibrating structure on the loads being applied by walking pedestrians. Presentation of all of the individual results would not be possible within this paper, however mean values for some of the more important recorded parameters are included in Table 3.

5. Analysis and discussion

The aim of the work described in this paper is to investigate the interaction between pedestrian loading and the vibration characteristics of flexible structures. To this end, the following discussion will examine how the loading affects the vibration of the structure and also how the structural vibration, in turn, alters the nature of this loading.

5.1. *Influence of pedestrians on the vibration response of the bridge-pedestrian system*

The presence of a pedestrian has an effect on the natural frequency of vibration of the flexible bridge. Table 4 shows how the natural frequency of the first vertical mode of vibration can be altered by the inclusion of an 82 kg person (P3) standing at midspan, and 3 individual pedestrians walking across the bridge respectively. For comparison purposes, the bridge configuration considered here is the 8.0 m span, as this gives the most flexible arrangement. The results are from 'fast' walking trials as these give the pacing frequencies closest to the bridge natural frequency. This particular combination was chosen as the extreme case.

Table 1
Design codes approach to pedestrian-induced vibration

Design Code	Satisfactory Fundamental Frequency (Hz)		Limits on Accelerations (m/s^2)	
	Vertical	Horizontal	Vertical	Horizontal
AASHTO	>3.0	>1.3	No limit specified	
Eurocode 5	>5.0	2.5	0.7	0.5 or 0.2
Eurocode 0	>5.0	2.5	0.7	0.5 or 0.2
SETRA	<1.7, >2.1	<0.5, >1.1		
Ontario Code	4.0 (implied)	4.0 (Implied)	Range	*
Canadian Code	4.0 (implied)	4.0 (Implied)	Range	*
Bro2004 (Swedish)	3.5	*	0.5	*
ISO 10137	N.A.	N.A.	$0.5\sqrt{fo}$ & 0.3	0.2
Hong Kong Code	5.0	1.5	$0.5\sqrt{fo}$	0.15
Australian Code	3.5	1.5	Range	*

Table 2
Age and Anthropometric data relating to test participants

Parameter	Overall (N = 26)		Female (N = 8)		Male (N = 18)	
	Mean	Std Dev	Mean	Std Dev	Mean	Std Dev
Age (years)	29.2	5.6	31.1	8.2	28.3	3.6
Height (m)	1.741	0.083	1.641	0.053	1.786	0.038
Mass (kg)	73.36	13.47	60.361	10.738	79.148	10.112

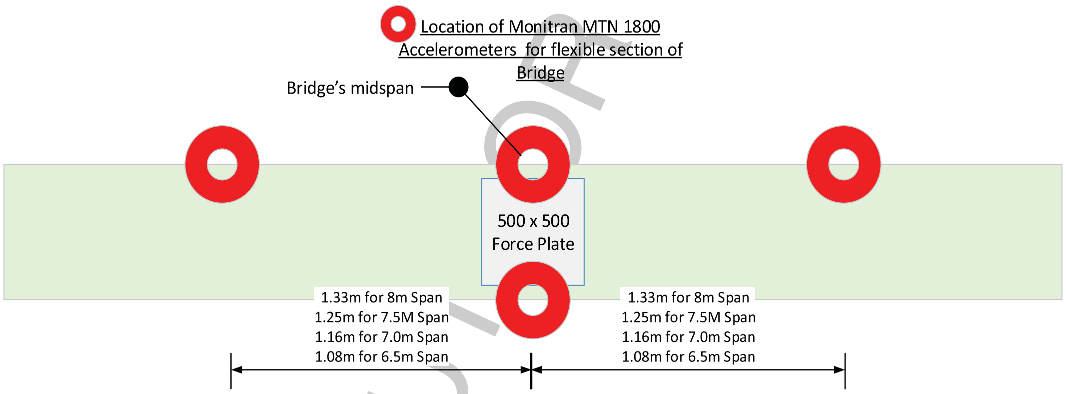


Fig. 4. Location of data acquisition equipment along flexible section of bridge.

It is evident from Table 3 that the pedestrian's presence on the bridge serves to alter the natural frequency of the combined bridge-pedestrian system, as would be expected. This is important in terms of modelling pedestrian loading as it reinforces the point that the loading cannot be applied by a moving or stationary point force alone. The authors present more detailed analysis of this effect elsewhere [19].

Of greater interest here, however, is the level of influence the pedestrian has on the natural frequency of the system. Prior to the walking trials, dynamic testing of the bridge was carried out. This yielded a vertical natural frequency of 2.97 Hz. When an 80 kg mass was placed at the centre of the bridge, this frequency reduced to 2.11 Hz. However, when the 82 kg person walked across the bridge, the frequency only reduced to 2.60 Hz. Further, the reduction in natural frequency was not proportional to the increased mass of the bridge when comparing three different pedestrians with mass in the range of 64 kg to 82 kg. These results indicate that the effect of the pedestrian on the natural frequency of the bridge-pedestrian system is not proportional solely to the mass of the pedestrian. Other characteristics, such as the pedestrian's own stiffness and damping properties should also be considered. This supports the argument that pedestrians cannot be accurately simulated as simply either moving forces or even moving masses with

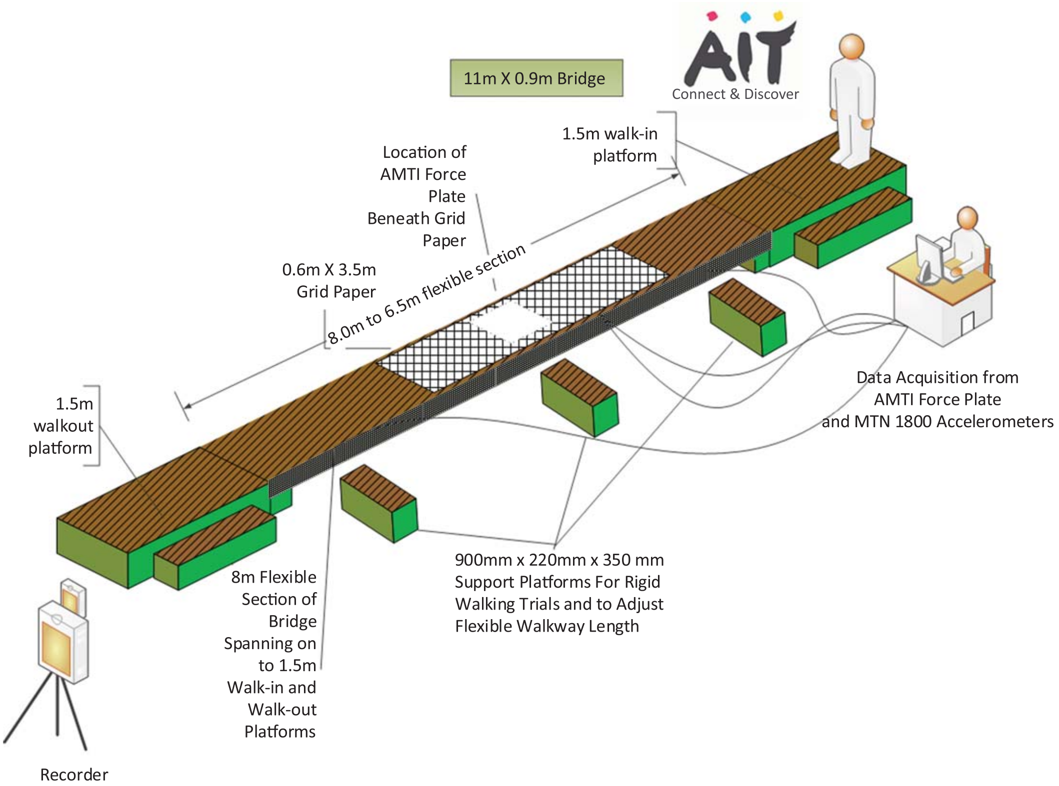


Fig. 5. Schematic representation of trial set-up (not to scale).

Table 3
Mean Values Recorded from Walking Trials *S = Nominal Slow Walking, N = Nominal Normal Walking, F = Nominal Fast Walking

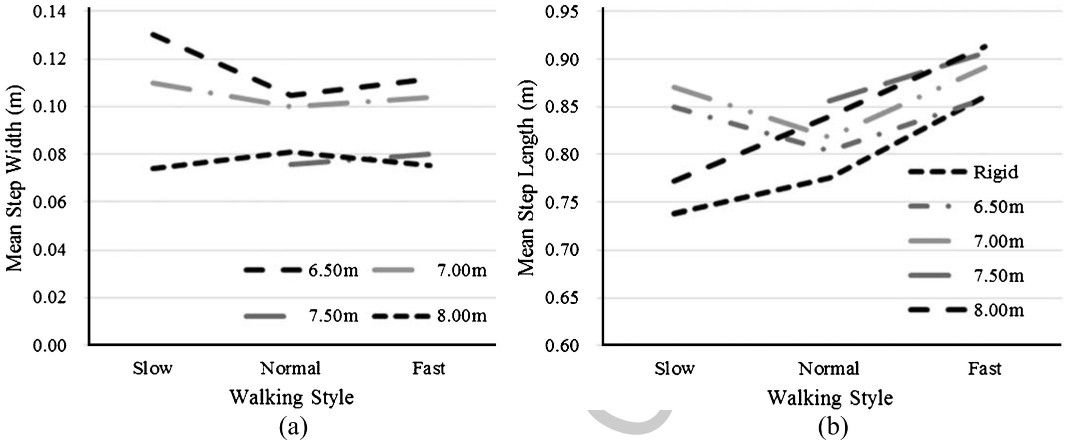
Span (m)	Rigid			6.5 m			7.0 m			7.5 m			8.0 m		
Style*	S	N	F	S	N	F	S	N	F	S	N	F	S	N	F
Step Length (m)	0.74	0.77	0.86	0.85	0.80	0.86	0.87	0.82	0.89		0.86	0.91	0.77	0.84	0.91
Step Width (m)	0.19	0.08	0.08	0.13	0.10	0.11	0.11	0.10	0.10		0.08	0.08	0.07	0.08	0.08
Pacing Frequency (Hz)	1.83	2.00	2.29	1.95	2.09	2.27	1.75	2.04	2.30		2.16	2.25	1.81	2.00	2.40
Weight (N)	752	726	726	752	770	770	692	701	701	692	738	741	719	719	719
Max. Vertical Force (N)		965	1113		1165	1335		1055	1186		1173	1272	960	1192	1196
Dynamic Force Ratio (DFR)		1.33	1.53		1.51	1.73		1.50	1.69		1.59	1.71	1.34	1.66	1.66
Change in DFR Flex to Rigid	–	–	–		0.14	0.13		0.13	0.10		0.20	0.12		0.25	0.08
Change in DFR Normal to Fast	–	–	0.15	–	–	0.15	–	–	0.12	–	–	0.08	–	–	0.00

(a) (b)

Fig. 6. Definition of Step Width (a) & Step Length (b).

Table 4
Influence of pedestrian loading

Bridge Loading	Empty	P3 (82 kg) standing at midspan	P10 (64 kg) Fast	P13 (65 kg) Fast	P3 (82 kg) Fast
Natural Frequency of 1st Vertical Mode (Hz)	2.97	2.11	2.77	2.55	2.60



(a) (b)

Fig. 7. Mean Step Width (a) and Mean Step Length (b) Versus Walking Style for each bridge span.

applied forces, but rather should be considered as altering the dynamic response of the bridge system due to their own dynamic stiffness properties.

5.2. Influence of bridge response on pedestrian loading

While the pedestrians present on the bridge certainly alter the vibration response of the structure, this response in turn can have an influence on the pedestrian's walking characteristics and hence the load applied to the bridge.

5.2.1. Bridge response vs step width & step length

Step width (w_s) is defined as the distance between the centre lines of the two feet, perpendicular to the plane of walking, as shown in Fig. 6(a). [23]. Archbold reports a review of current literature, citing references which yield values between 0.09 m and 0.19 m for adults, with no apparent link between subject height and step width [24]. In this case, the pedestrian's step width did not appear to change significantly as they progressed from slow to fast walking. Interestingly, though, the step width appeared to reduce as the bridge span (and hence vibration levels) increased (Fig. 7(a)). This could possibly reflect the pedestrian's attempt to improve their balance on a moving surface.

Step length is defined as the distance between a specific point on one footprint to the same point on the next (Fig. 6(b)). The step length increases as the pedestrian transitions from slow to fast walking for each of the test structure configurations, as would be expected. In general, the increase from slow to fast is in the order of 1-2%. However, for the 8.0 m span, the increase is 18%. One possible reason for this larger increase is that for the 8.0 m span, the mean fast pacing frequency is 2.40 Hz, which is approaching the resonant frequency of the structure. The test subjects may thus be altering their walking gait patterns to adapt to the behavior of the bridge.

5.2.2. Bridge response vs pacing frequency

The effect of the bridge response on the pacing frequency is presented in Fig. 8. This follows a similar trend to the effect on step length, but the influence of being close to the resonant frequency is more pronounced. For the 8.0 m span (again, the arrangement where pacing frequency comes closest to the natural frequency of vibration), the increase in pacing frequency from normal to fast walking is 20% – far higher than any of the other flexible configurations, where the mean increase is approximately 8%.

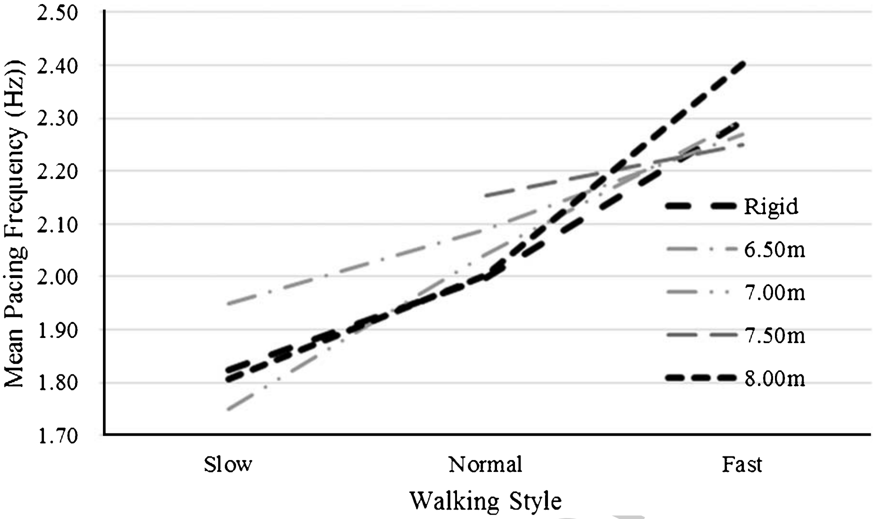


Fig. 8. Mean Pacing Frequency versus Walking Style for Each Test Arrangement.

Fig. 9. Mean Dynamic Force Ratio (DFR) versus Walking Style.

This sudden increase in pacing frequency, close to the vibration frequency of the bridge points towards an interactive effect, whereby the pedestrians pacing frequency tends towards that of the structure, once within a certain range.

5.2.3. *Bridge response versus dynamic vertical force*

The maximum vertical force recorded by the force plate at midspan is greater than the weight of a pedestrian, due to the addition of a dynamic component associated with the walking action of the pedestrian. The Dynamic Force Ratio (DFR) is the ratio of the peak vertical force to the static weight of the pedestrian. The mean DFR's for the normal and fast walking trials are contained in Fig. 9. As expected, the DFR increases with walking velocity in most cases. This is in line with what is reported by other authors, and in what is contained in design codes, whereby the force is proportional to the pacing frequency. The obvious exception to this relationship is for the 8.0 m span. In this instance, there is no increase in DFR from normal to fast walking, despite an increase in both step length and pacing frequency, which yield a 20% increase in mean walking velocity. In fact, there is a relative reduction in the DFR in this area where pacing frequency is close to the bridge response frequency.

It is theorized by the authors that this relative reduction is due to the fact the pedestrian alters their gait to “lock-in” with the vibrating bridge, effectively cushioning movements in the opposite direction. This relative reduction, in particular supports the approach of modelling pedestrians as moving spring mass damper systems, capable of ‘absorbing’ displacement of the bridge through their own inherent stiffness and damping system.

6. Conclusions

The authors have carried out walking trials on a specially constructed lightweight, flexible, laboratory scale footbridge with the objective of studying the influence of bridge vibration on the loading applied by crossing pedestrians. A total of more than 150 trials were carried out by 26 individual pedestrians, crossing the bridge at designated slow, normal and fast walking velocities. The trials were conducted over a range of support conditions for the bridge, providing both a rigid structure and flexible arrangements with spans of 6.50 m, 7.00 m, 7.50 m and 8.00 m respectively. This allowed for the study of various bridge response frequencies and a range of walking-induced excitation frequencies. The pedestrian movement was recorded by video analysis and through physical examination of their footfall traces. Applied vertical forces were directly recorded by a force plate at bridge midspan. The bridge response was measured using accelerometers along the span.

The results show that there is an apparent interaction between the vibration response of the bridge and the magnitude and nature of the vertical forces applied by the crossing pedestrians. The presence of the pedestrian on the bridge reduces the natural frequency of vibration of the bridge-pedestrian structure, but this is not just dependent on the mass of the pedestrian. Therefore, the stiffness characteristics of the pedestrian are important and must be considered in any model attempting to accurately simulate pedestrian loading on flexible structures.

The bridge response, in turn, influences the nature of the pedestrian loading, with the case where pacing frequency is close to the bridge’s natural frequency indicating increased changes in both the step length and pacing frequency.

The dynamic force component or the dynamic force ratio is seen to increase with walking velocity and pacing frequency in most cases. The exception to this is the 8.00 m span, where there is a relative decrease in DFR when going from normal to fast walking. It is the authors’ view that this is due to the ability of the pedestrian to regulate their gait in order to ‘cushion’ bridge movement in the opposite direction.

All of the above support the argument for considering pedestrians as dynamic spring mass damper elements rather than simply moving forces, or even moving masses with a dynamic force component, as currently recommended in many design codes.

The authors are currently updating and validating numerical models to replicate the results from these laboratory trials.

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